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## POSSIBLE ULTRASONIC EVIDENCE FOR THE EXISTENCE OF VORTEX-ANTIVORTEX PAIRS

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The attenuation of surface acoustic waves in the 700 MHz frequency range passing through 1000 Å NbN films has been measured on several NbN films. The films have a columnar structure where the columns are about 200 Å in diameter separated by 20 Å voids. In the superconducting state the attenuation does not follow the usual BCS curve. It appears to be composed of the sum of a BCS curve plus another curve which has a maximum below the superconducting transition temperature. The attenuation data may also be analyzed to yield an effective energy gap which is quenched at about one fifth of the BCS zero temperature energy gap. Tentatively, the Kosterlitz-Thouless vortex-antivortex model is used to determine the temperature dependence of an effective order parameter that yields reduced attenuation data which agree qualitatively with the experimental results.

The ultrasonic attenuation of surface acoustic waves in the 700 MHz range of frequencies has been measured as a function of temperature on nine separate superconducting NbN thin films. The results of these measurements were similar in that they could be decomposed into a BCS type curve and

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another bell-shaped curve having a maximum below the superconducting transition temperature. In this paper we shall compare our experimental results with the predictions for the attenuation which could be made using the Kosterlitz-Thouless vortex-antivortex model.

The nine films, sputter deposited<sup>1</sup> on YZ LiNbO<sub>3</sub> substrates, ranged in thickness from 0.02  $\mu$  to 0.3  $\mu$ . The films are columnar in nature. The columns are 85 Å in diameter separated in some places by 20 Å voids. Four-wire resistance measurements of the films were also done and these showed that the transition temperatures ranged from 14.2 K to 16.4 K, and that the normal state sheet resistivities ranged from a low of 70  $\Omega/\square$  to a high of 1100  $\Omega/\square$ . Thus a relatively wide range of sample properties has been measured. The deposition was on optically polished YZ LiNbO<sub>3</sub> substrates whose dimensions were  $1 \times 0.375 \times 0.030$ . The dimensions of the NbN films were approximately  $1.5 \text{ cm} \times 0.5 \text{ cm} \times \text{film thickness}$ . The interdigital electrodes (IDE) used to produce the surface acoustic waves had an interdigital spacing of about 2.6  $\mu$  and thus a resonant frequency of 700 MHz. The final configuration of the sample is shown in Ref. 2. A clear path was provided so that attenuation effects due to the substrate alone could be subtracted. Not shown in Ref. 2 are the four-wire resistance measurement bonding pads used for film resistance measurements.

To analyze the data, a linear component assumed not due to the superconducting properties was first subtracted. The result is shown in Fig. 1. This normalized graph has fea-

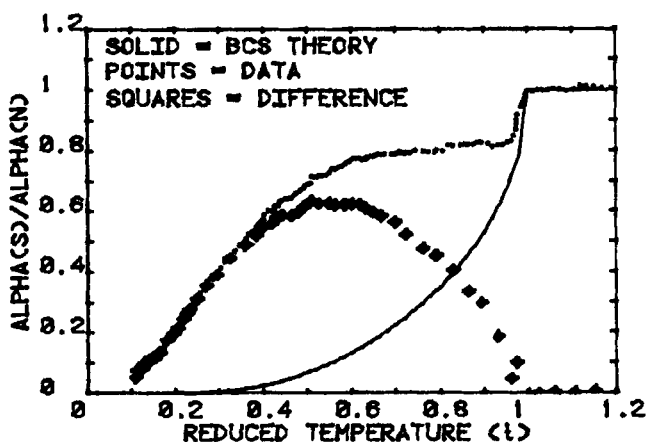


FIGURE 1 Normalized data with BCS prediction and difference for a 1000 Å film

tures that are typical of most of the samples measured. Details in exact shape and relative size of the step close to  $T_c$  varied somewhat. For the sample of Fig. 1, the normal state attenuation was  $6.5 \pm 0.15$  db and  $T_c$  was  $15.1 \pm 0.12$  K. Four-wire resistance of this film showed a normal state value of  $\sim 300 \Omega/\square$ , and this dropped to zero within 0.5 K of  $T_c$ . Also shown in Fig. 1 is the BCS prediction for the attenuation of ultrasonic waves in a superconducting medium having an energy gap at  $T = 0$  K of  $3.5 k_B T_c$ . The difference between the data and this prediction can be seen to have a maximum at about  $0.5 T_c$ . The analysis of the data can be continued by noting that the BCS prediction for the attenuation of ultrasonic waves depends on the temperature dependence of the superconducting energy gap parameter ( $\Delta$ ) in eq. (1).

$$\frac{\alpha_s}{\alpha_n} = \frac{2}{e^{\Delta/k_B T} + 1} \quad (1)$$

It was thus decided to find out what the temperature dependence of  $\Delta$  would have to be to produce the measured  $\alpha_s/\alpha_n$ . To this end, the above equation is solved for  $\Delta$  as a function of reduced temperature ( $t = T/T_c$ ):

$$\frac{\Delta}{k_B T_c} = \Delta(t) = t \ln \left( 2 \left( \frac{\alpha_n}{\alpha_s} \right) - 1 \right) \quad (2)$$

When the normalized data is inverted and substituted into this equation, the result is as shown in Fig. 2. This unusual deviation from the BCS prediction was striking and prompted a search for possible explanations.<sup>2</sup>

Experiments conducted on Al-Ge films by Deutscher and Rappaport<sup>3</sup> and on thin granular lead films by Hebard and Vandenburg<sup>4</sup> measured the film resistance as a function of temperature over the range of temperatures in which the films are superconducting. Their results showed characteristics explainable in terms of a double transition. This double transition has been ascribed to the predicted effects of vortex-antivortex pairs of flux lines. Since NbN films are known to possess a columnar structure, similar double transition behavior may be possible.

Kosterlitz and Thouless<sup>5</sup> have shown that some 2-dimensional superconducting systems may enter a lower energy state at a temperature below the superconducting transition temperature ( $T_c$ ) by forming vortex-antivortex pairs. Such

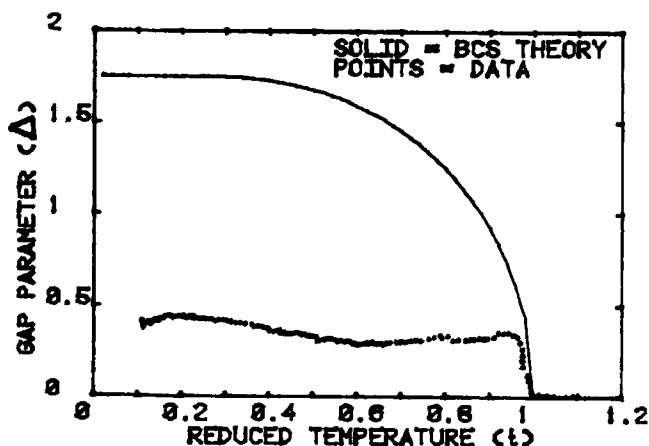


FIGURE 2 Effective order parameter from data and BCS prediction

pairs bound by a logarithmically varying interaction do not interact with applied currents until a dissociation temperature  $T_{KT}$  is reached. At this temperature, the bound pairs thermally dissociate into free vortices which may then interact with applied currents or fields through the mechanics of flux flow. This dissociation temperature has been found<sup>6</sup> to depend on the film resistivity, which for our films is relatively low. The resultant dissociation temperature should be greater than 98% of the BCS transition temperature. If as an approximation we say  $T_{KT} = T_c$ , the reduced temperature dependence for the pair density may be written<sup>7</sup>

$$n_d = \frac{1}{2\pi\xi^2} \exp\left(-\frac{1}{t}\right) \left(\frac{t}{(2-t)}\right) \quad (3)$$

where  $\xi$  is the coherence distance given by:

$$\xi^2 = \xi_0^2 (1-t+\delta)^{-1} \quad (4)$$

where  $\xi_0$  is the zero temperature coherence distance (between 30–40 Å for NbN) and  $\delta$  is the Maki-Thompson<sup>8,9</sup> parameter. The temperature dependence of this density reveals a maximum near 0.75  $t$ . A simple picture which assumes a short mean free path for normal state electrons in the superconducting

state would take the available normal state attenuation for a temperature below  $T_c$  and divide it equally among the existing vortices at the upper critical field  $H_{c2}$ . This would be the attenuation due to the presence of each flux line in a pair regardless of its orientation, since the interaction of the phonons with flux lines is independent of the orientation of the magnetic field. The cross-sectional area of a flux line is  $\pi\xi^2$  so that the number of flux lines per unit area at  $H_{c2}$  is  $1/\pi\xi^2$ . The normal state attenuation available is one minus eq. (1).

Multiplying this by the inverse of the number of flux lines and twice the density of flux pairs gives a model for the attenuation due to vortices for  $\ell \ll \xi$ :

$$\alpha_v = e^{-\frac{1}{t}} \frac{t}{2-t} \left( 1 - \frac{2}{e^{\Delta(t)/kT} + 1} \right) \quad (5)$$

A plot showing the sum of this and the BCS result is shown in Fig. 3.

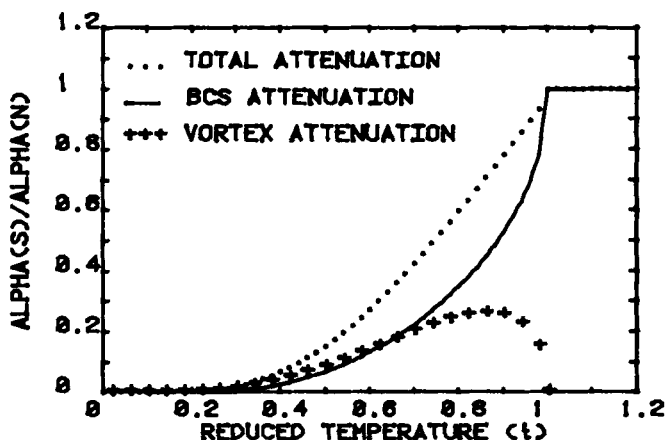


FIGURE 3 Attenuation from vortex theory when  $\ell \ll \xi$

As noted, eq. (5) applies for an electron mean free path short in comparison to the coherence distance. In a clean or pure type II superconductor the opposite is the case ( $\ell \gg \xi$ ). The electrons then would interact with a

spatial average of the order parameter. For this case, a one dimensional approximation where the vortices were taken to be all in a line was used. Here the order parameter went to zero at the center of each vortex with a linear dependence, so that the regions between vortices formed trapezoids of the order parameter. As the number of vortices increases the average value of the order parameter decreases. The temperature dependence of the order parameter does not change until the density reaches the point that vortices begin to "touch". This touching decreases the maximum value between vortices from its BCS value of  $\Delta_0$ . If the temperature at which the vortices touch is  $t_0$ , the temperature dependence of the average,  $\overline{\Delta}(t)$ , is:

$$\overline{\Delta}(t) = \Delta_0 \begin{cases} 1-n(t)/2n_0 & 0 < t < t_0 \\ n_0/2n(t) & t_0 < t < 1 \end{cases} \quad (6)$$

where  $n(t)$  is the temperature dependent density,  $n_0$  is the density at  $t_0$ . If  $\Delta_0 = 1.75 k_B T_c$ , the result for  $\overline{\Delta}(t)$  is shown in Fig. 4. If this  $\overline{\Delta}(t)$  is used in eq. (1), the calculated attenuation has the form shown in Fig. 5.

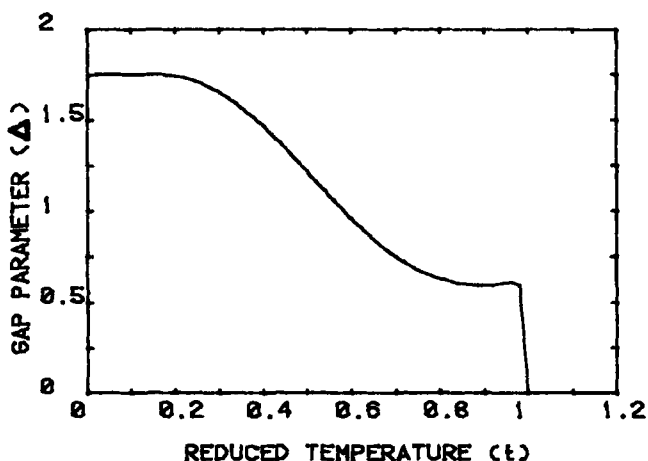


FIGURE 4 Simple triangular well model for  $\Delta$  when  $\ell \gg \xi$

As a more realistic approximation the linear sides of the vortices were replaced by the functional dependence of

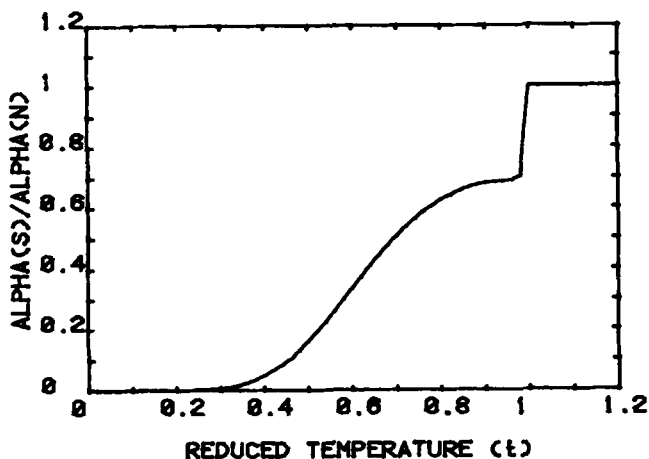


FIGURE 5 Attenuation calculated using  $\Delta$  from Fig. 4

the order parameter as derived from the Ginzburg-Landau theory.<sup>10</sup>

$$\Delta(x) = \Delta_0 \tanh \frac{x}{\sqrt{2} \xi(t)} \quad (7)$$

In this approximation, the average order parameter as a function of temperature is

$$\Delta_{AVE}(t) = \Delta_0 \begin{cases} 1 - 0.3374 \, n/n_0 & n < n_0 \\ \frac{n}{2n_0} \log(\cosh(\frac{2n_0}{n})) & n > n_0 \end{cases} \quad (8)$$

Here  $n_0$  is the density calculated from eq. (3) for  $t = t_0$ , the cut-off temperature at which the vortices begin to "touch". A plot of this is shown in Fig. 6, and the corresponding attenuation in Fig. 7.

It seems that the crude approximation of eq. (6) gives a closer approximation to the actual data. It may be that if the model is expanded to two dimensions so that the packing of vortices becomes important in determining the average value of the order parameter that this expanded model will fit the data even better. It should be mentioned that a much better fit to the data is possible if a zero tempera-

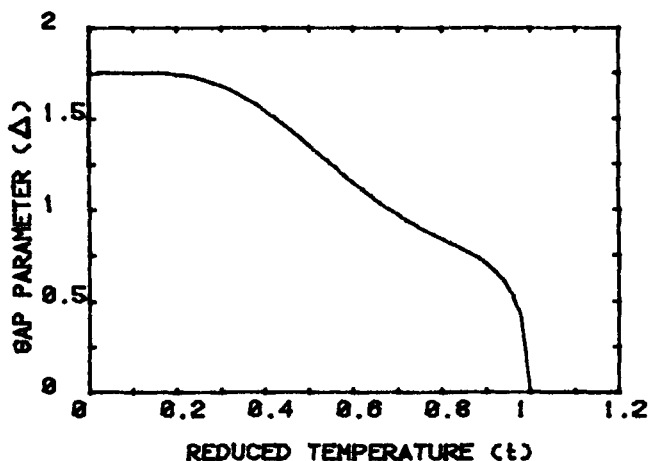
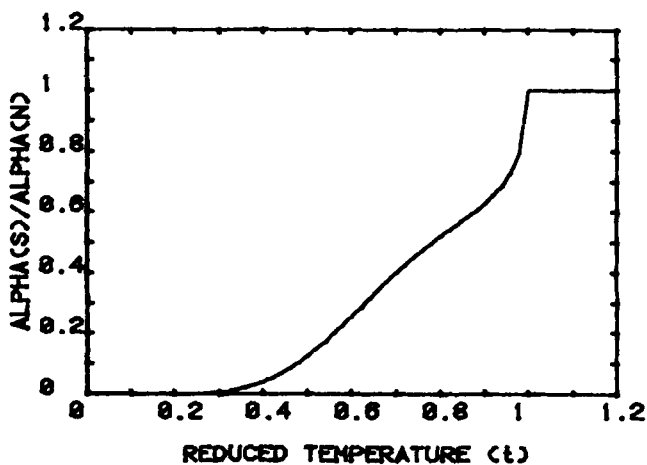


FIGURE 6 Order parameter using GL to define well shape

FIGURE 7 Attenuation calculated using  $\Delta$  from Fig. 6

ture order parameter of  $0.6 k_B T_C$  is used instead of the BCS result. Since no theoretical or experimental justification exists which could explain such a low value for  $\Delta_0$ , those results have not been included.

Two possible explanations for the attenuation of ultrasonic surface acoustic waves in superconducting NbN thin films have been briefly explored. The first model was

derived for sound waves propagating in a superconducting thin film where the mean free path of normal state electrons is assumed small compared to the coherence distance. The second was derived in the opposite limit. A comparison with the data shows that though the second model contains features similar to the data, both give an effect whose size is an order of magnitude less than the data.

#### REFERENCES

1. J. R. Gavaler, J. K. Hulm, M. A. Janocko, and C. K. Jones, *J. Vac. Sci. Tech.*, 6, no. 1, 177 (1969).
2. H. P. Fredricksen, M. Levy, J. Gavaler and M. Ashkin, *IEEE Trans. on Magnetics*, Vol. MAG-17, No. 1 (Jan. 1981).
3. G. Deutsher and M. L. Rappaport, *J. Physique Colloque*, C6 (1978) 581-582 (Supplement to *J. Phys. Paris* 39).
4. A. F. Hebard and J. M. Vandenburg, *Phys. Rev. Lett.* 44 (1980).
5. J. M. Kosterlitz and D. J. Thouless, *J. Phys. C.*, 6, 1181 (1973).
6. M. R. Beasley, J. E. Mooij and T. P. Orlando, *Phys. Rev. Lett.* 42, 1165 (1979).
7. L. A. Turkevich, *J. Phys. C.*, 12, L385 (1979).
8. K. Maki, in *Superconductivity*, edited by R. D. Parks (Dekker, New York, 1969), Vol. II, p. 1035.
9. R. S. Thompson, *Phys. Rev. B*, 1, 327 (1970).
10. See for example: St. James, Thomas, and Sarma, *Type II Superconductivity* (Pergamon Press, 1969).